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About verification of multilevel wavelet-based numerical method of local structural analysis for two-dimensional problems

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Abstract

The distinctive paper is devoted to verification aspects of multilevel method for local static analysis of two-dimensional structures based on the coupling of finite element method and discrete wavelet transform (FEM-DWT). Comparison of results, obtained by FEM-DWT and conventional finite element method (with the use of ANSYS Mechanical) is presented. The numerical results of the examples indicate that the proposed method provides exact results for selected regions, even in high level of reduction in wavelet coefficients.

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Keywords: Multilevel method; Numerical method; Wavelet-based method; Local static analysis; Two-dimensional problems; Discrete wavelet transform; Verification

1. Introduction

As known finite element method (FEM) is arguably the most powerful method known for the numerical solution of boundary problems and initial-value problems characterized by partial differential equations. Static structural analysis with the use of FEM leads to the resolving a system of linear algebraic equations with an immense number of unknowns [1,2]. Solution of such systems is apparently the most resource-consuming stage of the computing [3,4,5,6,7,8]. However, in many cases it is impossible or unreasonable to obtain corresponding numerical solutions

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for the entire structure and due to structural or loading conditions the location and approximate dimensions of critical and the most vital regions of the structure can be determined by designer. The stress-strain state in these regions may cause structural failure [9]. Sufficient accuracy in definition of this local stress-strain state with a smaller number of unknowns can be achieved with the use of so-called multilevel wavelet-based numerical method of local structural analysis [10,11,12]. The efficiency of the computational complexity of this method (based on coupling of finite element method and discrete wavelet transform (so-called FEM-DWT method), can be evaluated by the comparison of unreduced (n) and reduced (n_r) total number of degrees of freedom of the structure. Let N_{comp} and N^r_{comp} be values approximating computational complexity of the FEM and FEM-DWT, respectively,

$$N_{comp} = O(n^3)\,; \quad N^r_{comp} = O(n_r^3)\,. \tag{1}$$

Thus, the comparative reduction in size of the computation can be approximated by

$$N^r_{comp} / N_{comp} = O((n_r / n)^3)\,. \tag{2}$$

The distinctive paper is devoted to verification of FEM-DWT method. It is a rather efficient approach for evaluation of local phenomenon such as stress concentration or concentrated force. Furthermore, the proposed method allows qualitative and quantitative assessments of the degree of localization of various kinds of design factors and evaluation of the effect of each degree of freedom on the behavior of the structure. For verification and illustrating the efficiency of the FEM-DWT method in multilevel localization and reduction of problem size, a lot of numerical samples have been considered [10], several of them are presented below.

2. Analysis of Thin Plate Subjected to Self-weight

One dimensional local solution of thin plate with uniformly varying area subject to self-weight only, are studied by the FEM-DWT method (Fig. 1). The thickness of the plate is equal to $t = 0.02$ m. The module of elasticity of material of structure is equal to $E = 200$ GP, density is equal to $\rho = 85$ kN/m^3.

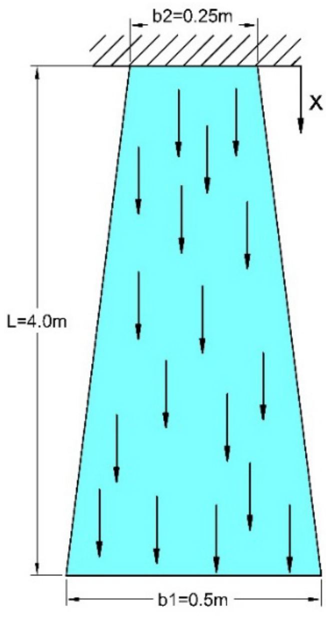


Fig. 1. Thin plate subjected to self-weight.

The first goal of the problem is obtain high accuracy local solution in the interval (region) $0 \le x \le 0.25L$ and the second one is compute maximum stress in the plate, by using the maximum degree of localization. Initial solution by FEM has been obtained by using 256 nodes ($M = 8$). The same discretization also used initially for local analysis. A comparison between the local solution (FEM-DWT method) and the FEM solution shows in Fig. 2. The local results obtained by FEM-DWT method agree well with the FEM results in the selected interval, even for high reduction in size of the problem. In this example the reduction has been imposed in one side of the stiffness matrix (i.e. last 0.75 part of the problem), and we obtained the maximum localization (minimum size of the problem) equal to 70 nodes with high accuracy results (Fig. 2a). Fig. 2b shows the comparison of maximum stress in the plate and its relative nodes for FEM solution and FEM-DWT solution.

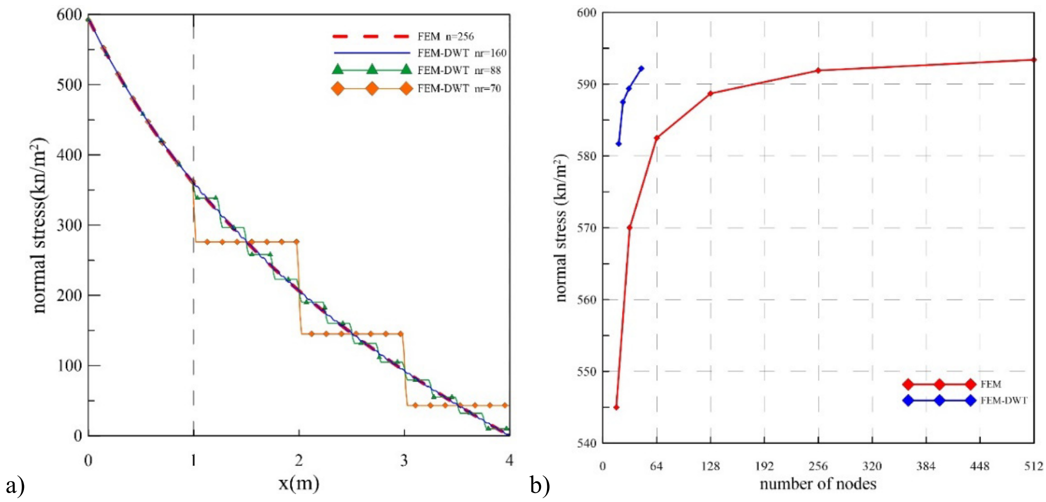


Fig. 2 Comparison of FEM and FEM-DWT results of plate analysis: a) normal stress; b) maximum stress.

As the results shows, with the use of FEM-DWT method, we can obtain the maximum stress in much more less number of nodes (size of governing equation) with acceptable accuracy by comparison of the high number of nodes in FEM. Table. 1 demonstrates the comparison of results of FEM and FEM – DWT for evaluating of maximum stress in the structure.

Table.1 evaluation of maximum stress in the plate by FEM and FEM-DWT.

Solution method	Number of nodes	Evaluated Maximum stress (KN/m^2)	Error[1] (%)
FEM	16	545.0	8.9
	32	570.0	4.12
	64	582.5	1.89
	128	588.7	0.82
	256	591.9	0.27
	512	593.5	0.0
FEM-DWT	19	581.7	2.0
	24	587.5	1.02
	31	589.4	0.7
	46	592.2	0.22

1. Error with respect to FEM solution by 512 nodes

3. Thin Cantilever Plate Subjected to Distributed Load

Thin cantilever plate, subjected to distributed load $q = 1000$ kN/m, are studied by FEM-DWT method (Fig. 3). The thickness of the plate is equal to unit and its module of elasticity and Poisson ratio of material is equal to $E = 200$ GP and $\nu = 0.3$, respectively. In order to obtain FEM solution, the plate is discretized by 4-nodes isoparametric quadrilateral elements. The main goal is to obtain high accuracy local solution in the region $0 \le x < 0.3$ and $0 \le y < 0.5$.

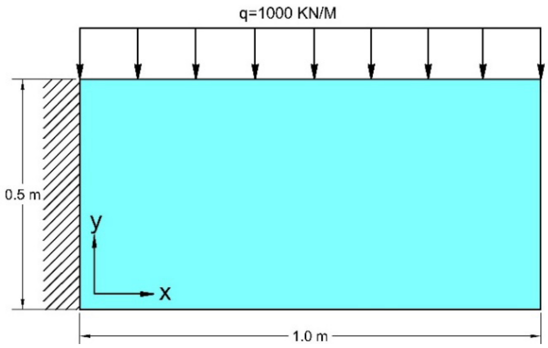


Fig. 3. Thin cantilever plate subjected to distributed load.

Initial FEM solution was obtained with $n = 512$ (n is the total number of nodes) and then three types of reduction were imposed. First of all, localization by node was performed and the size of the problem reduced to $n_r = 332$ (n_r is the reduced total number of nodes). Then localization by degree of freedom along x ($n_{r,1}$) and y ($n_{r,2}$) directions was performed as well. With respect to structure and loading condition, the maximum variation of stress occur under the load. Therefore we selected cross-section $y = 0.5$ m for comparisons of the evaluated stresses along x direction of the plate (Fig. 4, 5). Corresponding results agree well. The produced turbulence near the localization line is caused by nearing reduced node. It can be eliminate by matching of the border of reduction zone and final unreduced node.

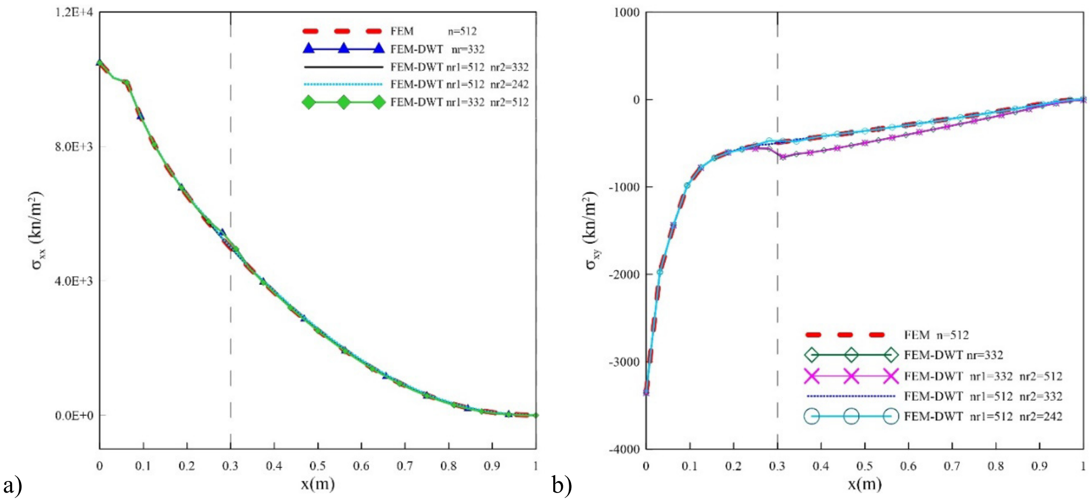


Fig. 4. Comparison of results for $y = 0.5$ m and along the x direction: a) σ_{xx} ; b) σ_{xy} .

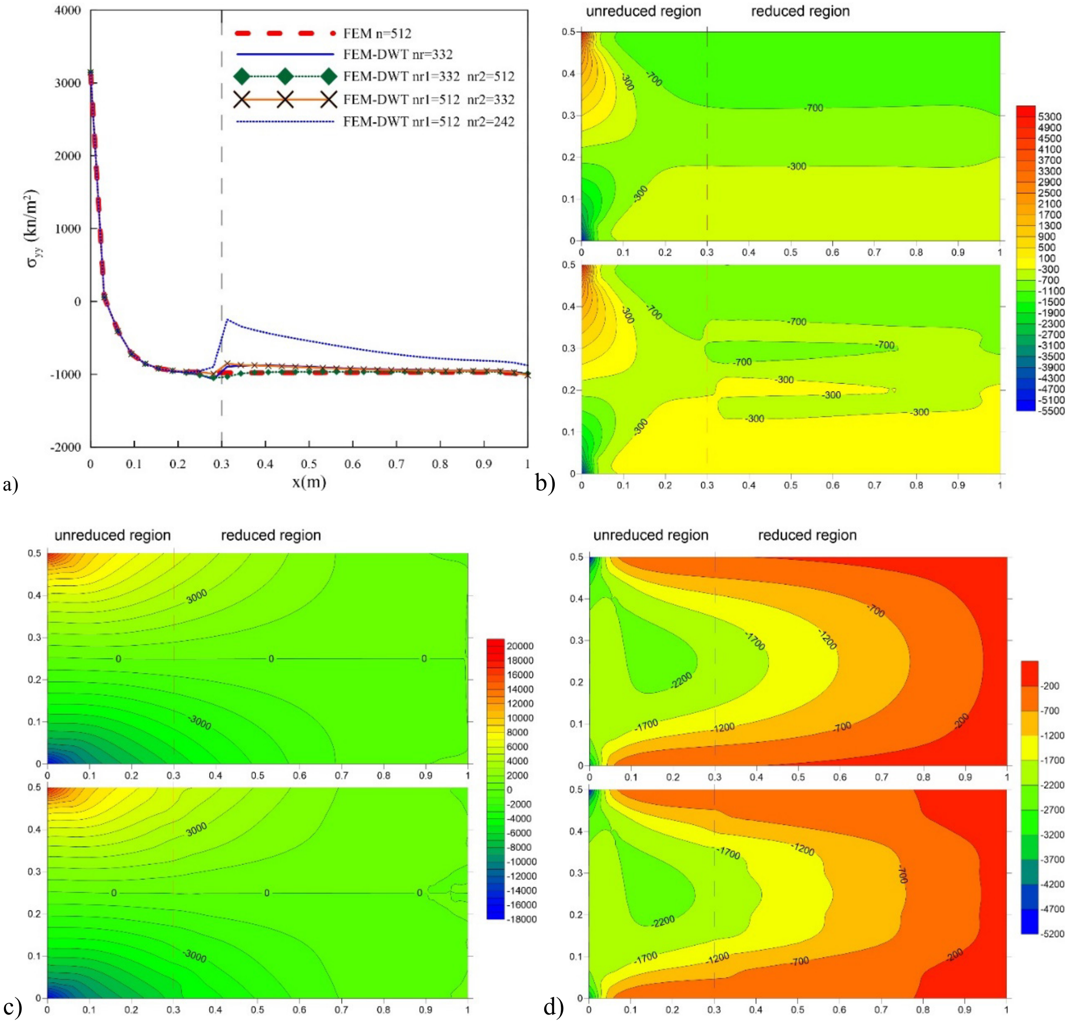


Fig. 5. Comparison of results: a) σ_{yy} for $y = 0.5$ m and along the x ; b) σ_{xx} – FEM and FEM-DWT ($n_r = 332$) solutions; c) σ_{xy} – FEM and FEM-DWT ($n_r = 332$) solutions; d) σ_{yy} – FEM and FEM-DWT ($n_r = 332$) solutions.

4. Thin Constrained Deep Beam Subjected to Concentrated Force.

Thin deep beam, subjected to concentrated force $P = 1000$ kN, are studied by the FEM-DWT method (Fig. 6). The thickness of the plate is equal to unit and its module of elasticity and Poisson ratio of material is equal to $E = 20$ GP and $\nu = 0.3$, respectively. In order to obtain FEM solution, the plate is discretized by 4-nodes isoparametric quadrilateral elements. With respect to problem constrains and loading condition, the localization has been imposed in two direction. Therefore, the goal of the problem is obtain high accuracy local solution in the region $0.5 \le x < 1.0$ and $0.5 \le y < 1.0$.

Initial FEM solution was obtained with $n = 256$ and then three types of reduction were imposed. First of all, localization by node was performed and the size of the problem reduced to $n_r = 160$. Then localization by degree of freedom along x ($n_{r,1}$) and y ($n_{r,2}$) directions was performed as well.

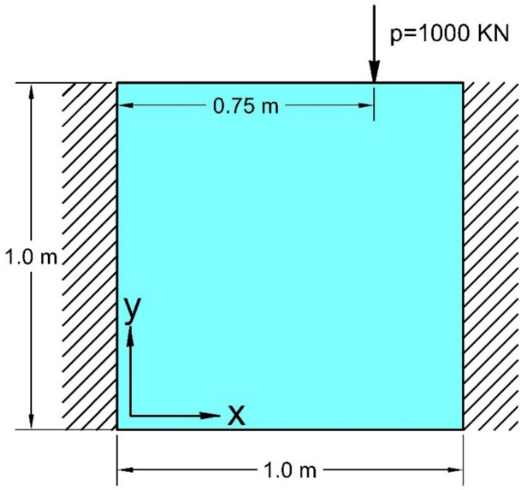
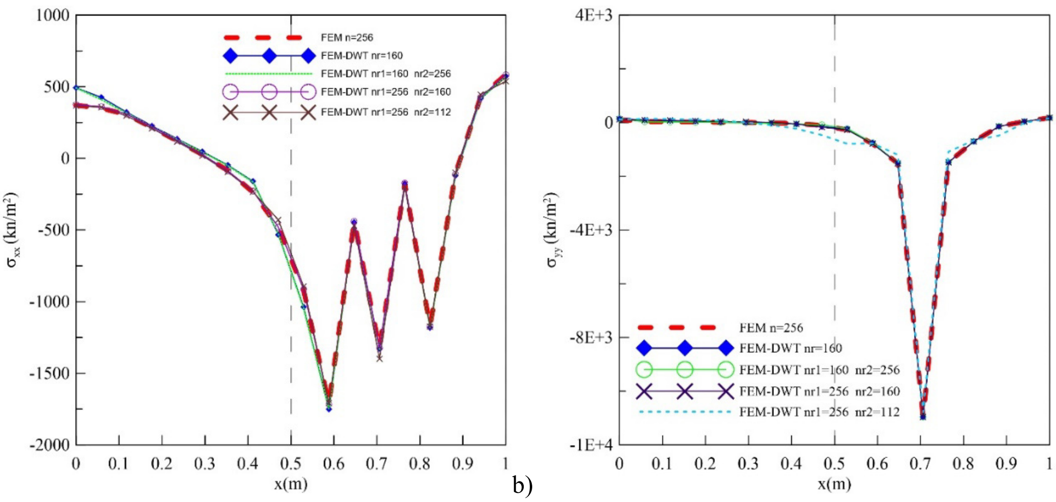


Fig. 6. Deep beam subjected to concentrated force.

Cross-sections $y = 0.5$ m and $x = 0.1$ have been selected for comparisons of the evaluated stresses along x and y direction of the plate, respectively (Fig. 7, 8) . Corresponding results agree well. The produced turbulence near the localization line is caused by nearing reduced node. It can be eliminate by matching of the border of reduction zone and final unreduced node.



a) b)

Fig. 7. Comparison of results: a) σ_{xx} for $y = 0.5$ m and along the x ; b) σ_{yy} for $y = 0.5$ m and along the x .

Conclusion.

Thus, we can conclude that the results of analysis obtained by the FEM (ANSYS Mechanical 15.0) and FEM-DWT method generally agree well with each other. It was confirmed that FEM-DWT could be used for the high-accuracy local analysis of the most critical, vital, potentially dangerous areas of structure in terms of fracture (areas of the so-called edge effects), where some components of solution are rapidly changing functions and their rate of change in many cases requires immense number of unknowns for global analysis.

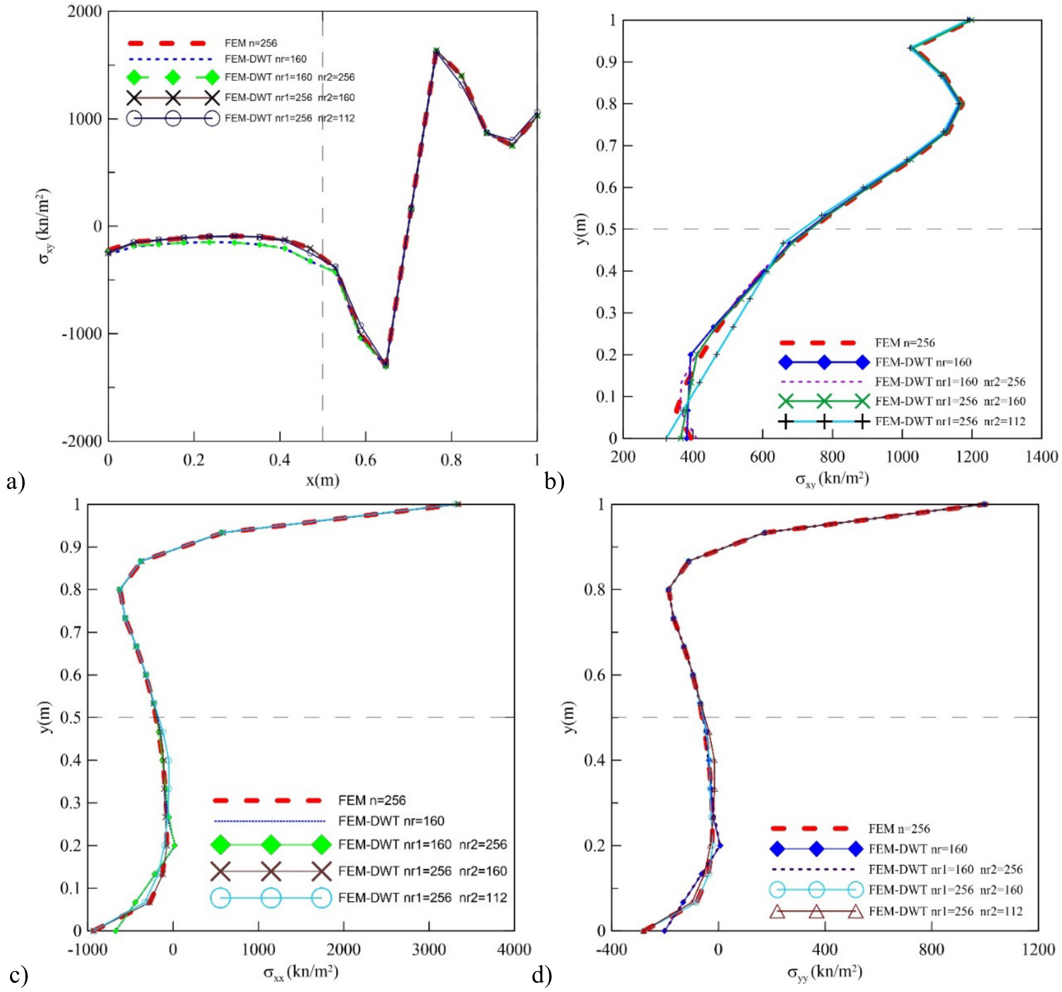


Fig. 8. Comparison of results: a) σ_{xy} for $y = 0.5$ m and along the x ; b) σ_{xy} for $x = 0.1$ m and along the y ;

c) σ_{xx} for $x = 0.1$ m and along the y ; d) σ_{yy} for $x = 0.1$ m and along the y .

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